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EXPERIMENTAL VERIFICATION OF DYNAMIC MODELS

by

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This is an abstract of SAE paper No. 781036 which is not yet available in its final form. The author apologies for this and will provide a sign up sheet at the 1978 Aerospace Meeting so that a copy can be forwarded to you. Copies can also be obtained by corresponding to the above address.

1. Introduction

Verifying the adequacy of structures and equipment to meet structural criteria requires both theoretical and experimental analysis. This is particularly true for the seismic analysis of structures and equipment in nuclear power plants and other essential facilities. Herein are presented several case studies of how combined theoretical and experimental studies have been used to qualify equipment. These studies may involve rather simple field and in-situ test methods or elaborate mathematical analysis of both theoretical models and experimental data and optimization of models to best fit experimental data.

2. Nature of Testing

Testing of structures and equipment can be divided into "proof testing" and "identification testing." Proof testing exposes the test object to forces and ground motions anticipated during the design seismic event (i.e., "mission loads"). Many types of equipment, weighing up to several tons, can be proof tested on shake tables. There has been a gradual evolution in these tables over the last ten years from unidirectional sinusoidal devices to bidirectional arbitrary input tables of great sophistication. Proof tests of large pieces of equipment and full scale structures can also be carried out by detonating high explosives in the soil nearby the structure. Charges as little as 40 kg and as great as 80 tons have been used. The resulting ground motion can be manipulated, within certain limits, to simulate desired earthquake design ground motions.

Identification testing is used to verify if theoretical models are adequate or to aid in formulating theoretical models. In these cases the nature of testing need not be "earthquake like" but rather just excite the structure so as to betray its actual dynamic properties. Testing methods include ambient excitation, impact, snapback, sinusoidal tests, and transient excitation. Generally the objective is to identify resonant frequencies, mode shapes, and dampings. At this point several parameter identification techniques are available to estimate these "eigenparameters." These eigenparameters, in themselves, may be the end product, sufficient to verify a computer model or to demonstrate compliance with design standards. On the other hand the eigenparameters may serve as the starting point for additional modeling effort and modification.

3. Model Modification

Once the eigenparameters are estimated the analyst may ask himself "how should I change my model to more closely reproduce the experimental data?" This can be carried out by simple heuristic "trial and error" methods or by more sophisticated mathematical techniques. Of the latter Bayesian Identification is of particular elegance and power.

The objective of Bayesian Parameter Identification (BPI) is to find a set of optimal model parameters which simultaneously minimizes the difference between measured and predicted response and between initial (a priori) parameters and final optimal parameters. This dependence on a priori parameter estimates is justified on two grounds. First, experimental data often do not uniquely define the model parameters and additional constraints are required to choose a unique set. Second, one assumes that the analyst's a priori choice of model parameters is a reasonable one based on his considered judgment, previous results, and preliminary data. Consequently, it is reasonable to introduce additional constraints by choosing the set of optimal parameters that differs in some least way from the initial estimates. These minimum criteria are least square in nature and are weighted to allow more certain data and a priori parameters to control the optimal parameter selection more than less certain ones.

Consider an example of a single-degree-of-freedom oscillator. The analyst has estimated its mass at 1.0 kg with an uncertainty of ± 0.32 kg. The stiffness has been estimated at 1.0 N/M ± 0.55 N/M. The predicted resonant frequency is thus 1.0 radian/second, but is measured at 0.9 radian/second with experimental error of ± 0.22 radians/second. What is the best estimate of mass and stiffness based upon these data? Clearly the problem is undetermined. A stiffness of 0.81 and mass 1.00 is a solution. A stiffness of 1.0 and mass 1.23 is a solution. For that matter a stiffness of 100.0 and a mass of 123.0 is a solution. The latter case is unreasonable based on the analyst's estimates. No cases account for the possible error in the data.

The BPI technique seeks to introduce uniqueness and account for a priori estimate error and data error by minimizing the following error as a function of the model parameters:

$$E(m_o, k_o) \equiv \frac{(k - k_o)^2}{\sigma_k^2} + \frac{(m - m_o)^2}{\sigma_m^2} + \frac{(w_o - w_m)^2}{\sigma_w^2}$$

where m = a priori mass estimate;

k = a priori stiffness estimate;

m_o = optimal value of mass;

k_o = optimal value of stiffness;

w_m = measured value of resonant frequency;

w_o = optimal value of resonant frequency corresponding to m_o and k_o;

σ_k = uncertainty in a priori stiffness estimate;

σ_m = uncertainty in a priori mass estimate; and,

σ_w = uncertainty in measured resonant frequency.

The model parameters, m_o and k_o, are related to the measured parameters, w_I through the model

$$W_o = \sqrt{\frac{k_o}{m_o}}$$

thus

$$E(m_o, k_o) = \frac{(k - k_o)^2}{\sigma_k^2} + \frac{(m - m_o)^2}{\sigma_m^2} + \frac{\left(\sqrt{\frac{k_o}{m_o}} - W_m\right)^2}{\sigma_w^2} .$$

This error function can be minimized either by setting its partial derivative with respect to m_o and k_o to zero and solving the resulting nonlinear equations, or by numerical techniques. In either case the solution is:

Parameter	A Priori or Measured Value	Optimal Value	% Difference
Mass	1.00 ± 0.32	1.030	3.0
Stiffness	1.00 ± 0.55	0.895	11.0
Resonant Frequency	0.90 ± 0.22	0.932	3.6

As can be seen, a "happy medium" has been found — changing the stiffness more than the mass (as its uncertainty was greater) and matching the resonant frequency closer than the average of the model parameter changes (as its uncertainty was least).

This technique can be generalized to treat any number of measured and model parameters. Linearization of the criteria function yields a particularly simple algorithm.

4. Case Studies

Two major structures and two pieces of equipment are discussed here. Soil-structure interaction occurs where a heavy and relatively rigid structure such as a nuclear power plant containment is founded on compliant soil or rock. This interaction significantly effects the response of the structure and consequent input to internal equipment. Its analysis is complicated by the nonlinear properties of soils and the difficulty of measuring these properties.

We are currently engaged in two research efforts to verify nonlinear modeling techniques for soil-structure interaction. One involves an actual nuclear power plant in the Federal Republic of Germany and the other a series of tests on models for the Electric Power Research Institute. The German tests were performed after several independent groups modeled the containment. Eccentric mass vibrators were used to determine the resonant frequencies and mode shapes at levels of response reaching .01 g. Bayesian techniques were used to optimize the models to more closely match experimental data. The comparison of models to the data revealed the advantages and deficiences of the various techniques used. Subsequent tests will use longer vibrators and blast techniques to reach 0.2 g response and more fully investigate nonlinear effects.

The tests for EPRI involve several containment models from 1/48 to 1/8 of full scale. Up to 80 tons of explosives were used to simulate strong earthquake ground motion. Nonlinear finite element and finite difference models were developed and used to predict response of the containment and soil. Vibrators were also used to evaluate dynamic properties at various levels of response. Remarkable nonlinear softening was observed and is leading to reformulation of the nonlinear soil-structure interaction problem. Related tests are using dynamic soil testing methods to evaluate nonlinear soil parameters in-situ. Bayesian techniques are used for data reduction.

Complex equipment must often be tested to ensure confidence in its seismic capacity. Electrical distribution equipment including the switch gear and the instrumentation cabinet discussed below fall into this category. The 230 KV/2,000 AMP disconnect switch was proof tested on a shake table. Three of these one-ton switches are field mounted on a common frame. As the dynamics of this frame will affect the input motion to these switches both a finite element model and field testing was used to identify the frame dynamics. Here rather surprising results were obtained that underlined the need for careful analysis and confirmatory testing. The finite element model indicated a first frame frequency of 4.0 Hz. In the field, however, the frequency was measured at 1.9 Hz. This difference was found to be caused by differences in assumed and actual base plate mounting conditions. Incorporations of these conditions into the theoretical model brought the theoretical frequency down to 1.8 Hz. This model was then used to predict the input to the switch gear. This input was then used on the shake table. The significance of the frame dynamics error would have been great, if uncorrected, as the switch gear possessed resonances near 4.0 Hz.

An electrical instrumentation control panel was tested by in-situ methods to verify a computer model and to provide guidance to shop modifications to meet resonant frequency criteria. A finite element mathematical model was used to predict global frame frequencies. The first mode was predicted at 30.5 Hz. To confirm the frame modes and establish that all panel modes were above 30 Hz (a design criteria) in-situ tests were performed with a small eccentric mass vibrator. The first frame mode was found at 22.6 Hz and attributed to flexible mounting conditions and floor flexibility. The finite element model was modified accordingly. Several panel modes were found below 30 Hz but additional braces were installed immediately and retesting confirmed that frequencies were elevated above 30 Hz.